FURTHER KINEMATICS

WHAT DO I NEED TO KNOW

1. To solve problems involving constant acceleration in 2 dimensions, use the SUVAT equations with vector components where
u is the initial velocity
a is the acceleration
v is the velocity at time t (t is a scalar)
r is the displacement at time t

2. To solve problems involving variable acceleration in 2 dimensions, use calculus with vectors by considering each function of time (the vector component) separately.

3. When integrating a vector for a variable acceleration problem, the constant of integration, c, will also be a vector.

4. To find constants of integration, look for initial conditions or boundary conditions.

5. Displacement, velocity & acceleration can be given using *i*-*j* notation, or as column vectors.

FORMULAE

The formula to find **the position vector, r,** of a particle starting at position $\boldsymbol{r}_0$ that is moving with constant velocity **v** is

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$$

Constant acceleration vector equations:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

Calculus for variable acceleration:

Velocity, if displacement is a function of time:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}$$

$$\int (\boldsymbol{v})\,\mathrm{d}t = \boldsymbol{s}$$

Acceleration, if velocity is a function of time

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{s}}{\mathrm{d}t^2}$$

$$\int (\boldsymbol{a})\,\mathrm{d}t = \boldsymbol{v}$$

DOT NOTATION & DIFFERENTIATING VECTORS

Dot notation is a shorthand for differentiation with respect to time.

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad \ddot{x} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} \qquad \ddot{y} = \frac{d^2y}{dt^2}$$

To differentiate a vector quantitiy in the form f(t)i + g(t)j, differentiate each function of time separately.

If $\boldsymbol{r} = x\mathrm{i} + y\boldsymbol{j}$, then $\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \dot{\boldsymbol{r}} = \dot{x}\mathrm{i} + \dot{y}\boldsymbol{j}$ and $\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = \ddot{\boldsymbol{r}} = \ddot{x}\mathrm{i} + \ddot{y}\boldsymbol{j}$

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